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118 EXERCISES.

where V is the velocity of the body in its orbit, k the constant of the system, ϕ the angle between the tangent to the new orbit and the prolongation of the corresponding radius vector (r), r_0 the radius vector for the vertex of the old orbit, and p, a, and e are the semi-parameter, semi-major axis, and eccentricity of the new orbit. For the given point we have, as may easily be seen,

$$\begin{aligned}
r &= 2m, \\
\psi &= 30^{\circ}, \\
r^2 \sin^2 \psi &= m^2;
\end{aligned}$$

whence, by substitution in (a), we see that the new orbit will be an hyperbola, in which p = 2m, a = -m, e = 1 3. [Ormond Stone.]

EXERCISES.

26

DETERMINE the maximum right cone B inscribed in a given right cone A, the vertex of B being at the centre of the base of A. [O. Root, Jr.]

27

The number of points common to three surfaces of the *m*th, *n*th, and *p*th degrees being in general *mnp*, find the co-ordinates of all the real and imaginary points of intersection of the three surfaces, $y^2 + x^2 + z^2 = 4x$, $x^2 = yz$, and $y^2 = x^4$.

[H. A. Newton.]

28

GIVEN the perpendicular, median, and bisector issuing from one and the same vertex of a plane triangle and terminating in the opposite side, to construct the triangle and determine a formula for its area.

[Marcus Baker.]

29

GIVEN a pair of points A,B, if C,D are such that $OD \cdot OC = OA^2 = OB^2$ and AOC = AOD, prove that the pair A,B bears similar relation to the pair C,D. Show, also, the existence of a pair E,F which bears the same relation to each of the pairs A,B and C,D.

[Wm. Woolsey Johnson.]

30

Investigate formulæ for the logarithms of 13, 19, and 73 in terms of the logarithms of primes less than 13 and of (N+1/N) where N=132495; 262143; 274625. [F. H. Loud.]

31

In the triangle ABC draw AD to the point D in BC; then will

$$AB^2$$
. $DC + AC^2$. $BD = BC$. $AD^2 + BD$. DC . BC .

[J. R. Spiegel.]

32

A TRIANGLE PQR is inscribed in triangle ABC. Determine the ratios in which P, Q, R divide the sides BC, CA, AB in order that AQR, BRP, CPQ may be respectively $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, of ABC. [W. M. Thornton.]

SELECTED .- ALGEBRAIC EQUATIONS.

33

The cubic $ax^3 + 3bx^2 + 3cx + d = 0$ has one or three real roots as its discriminant

$$D = (ad - bc)^{2} - 4(b^{2} - ac)(c^{2} - bd)$$

is positive or negative. If D is zero, two roots are in general equal each to

$$\sqrt{\frac{c^2 - bd}{b^2 - ac}}.$$

34

Solve by quadratics the equations below and interpret their roots geometrically:—

$$x^{4} - x^{3} \sqrt{15} + 4x^{2} - 1 = 0,$$

$$x^{4} - x^{3} \sqrt{3} - 2x^{2} + 2x \sqrt{3} - 1 = 0.$$
35

Solve by quadratics the quartic

$$x^4 + ax^3 + bx^2 + ad^{\frac{1}{2}}x + d = 0.$$

36

Solve the cubic

$$\sqrt[3]{x} + \sqrt[3]{x - 7} = \sqrt[3]{3x + 3}.$$

by eliminating its commensurable root.

I 20 EXERCISES.

37

If a, b, c, d, e be the roots of the quintic

$$x^5 + px^3 + qx^2 + rx + s = 0$$

find the values of

$$a^{2} (de + eb + bc) + b^{2} (ea + ac + cd) + c^{2} (ab + bd + de) + d^{2} (bc + ce + ea) + e^{2} (cd + da + ab).$$

Show that among any three roots a, b, c of a biquadratic, as $x^4 + px^2 + q$ = 0, this relations holds:—

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}.$$
CONICS.
39

On a Chord MN of an ellipse as diameter a circle is drawn which cuts the curve in P and Q. Show that the ratio of the intercepts of MN and PQ on the axis major is $e^2 : 2 - e^2$.

40

The vertex of an hyperbola and one asymptote is fixed. Find the locus of the focus.

41

FIND the equations to the conics which cut the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

orthogonally at all their common points.

42

From two points on Ox, equidistant from O, tangents are drawn to the conic $ax^2 + 2hxy + by^2 = 2x$.

Find the locus of their points of concourse.

43

THE POLARS of points on one of two equal circles relative to the other envelop a parabola.

44

Find the locus of the points from which tangents drawn to $Ax^2 + By^2 = 1$ are parallel to conjugates of $ax^2 + 2hxy + by^2 = 1$.